An evaluation of interpolation methods for Mars Orbiter Laser Altimeter (MOLA) data

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Abstract

The Mars Orbiter Laser Altimeter (MOLA) instrument on the Mars Global Surveyor (MGS) spacecraft has returned a large amount of data on the topography of Mars. It is possible to generate high-resolution Digital Elevation Models (DEMs) from this data by employing data interpolation techniques. Four interpolation algorithms were selected for testing on MOLA data: Delaunay-based linear interpolation, splining, nearest neighbour, and natural neighbour. These methods were applied to the MOLA data of Korolev crater for qualitative analysis. In addition, a DEM of a part of Iceland was used for quantitative testing by simulating MOLA data acquisition, interpolating that data, and then calculating the mean absolute error (MAE) between the interpolated and original DEM. Execution speeds were measured for the four algorithms. The natural neighbour method proved superior both quantitatively and qualitatively to other methods tested, but is relatively slow computationally.

1. Introduction

The Mars Orbiter Laser Altimeter (MOLA) instrument on the Mars Global Surveyor (MGS) spacecraft acquired a large topography dataset from the orbit of Mars. MOLA fired 1064 nm laser pulses at the surface at a rate of about 10 per second (Zuber *et al*. 1992). When the laser strikes the surface, some fraction of the laser energy is backscattered in the direction of the spacecraft, and range is calculated from round-trip travel time. From September 1997 to June 2001, over 600 million data points have been collected. The MGS spacecraft is in a near-polar orbit with an inclination of 93° (Albee *et al*. 1998), so the density of coverage by MOLA increases with latitude.

Currently available digital elevation models (DEMs) of Mars, known as Experiment Gridded Data Records (EGDRs), are constructed by taking the median observed topography within a specified degree area. At the time of writing, the highest resolution EGDR is 1/32 degrees per pixel (Smith *et al*. 2001).

However, using data interpolation techniques instead of calculating median elevations can result in higher-resolution DEMs. But, it should be noted that most common interpolation algorithms were formulated to work with randomly distributed data (Watson 1992), and give visible artefacts when applied to MOLA tracks. The data points within each MOLA track are regularly spaced, and the tracks themselves are either nearly parallel to each other or intersect each other at a latitude-dependent angle. The challenge is to find an algorithm that minimizes visual artefacts and is quantitatively as accurate as possible.

The goal of this project was to test several common interpolation techniques, namely Delaunay-based linear interpolation, splining, nearest neighbour (also known as inverse distance weighting), and natural neighbour. These techniques were applied to MOLA data for qualitative testing. In addition, quantitative testing of simulated MOLA data obtained from a complete DEM was performed.

2. Interpolation Methods

2a. Linear Interpolation

Linear interpolation performs a Delaunay triangulation of a planar set of data points. After the irregularly gridded data points have been triangulated, the surface values are interpolated to a regular grid. Given the values of some observable (f_i, $i = 1, \dots, 3$) at the nodes of a Delaunay triangle (x_i, y_i, $i = 1, \dots, 3$) , the interpolated value at any point (x, y) interior to the triangle is given by

$$\mathrm{f(x,y)} = \sum_{i=1}^{3} \phi_i(x, y) f_i, \qquad (1)$$

where $\phi_i(x, y)$ is a 2-D basis function which varies linearly from a value of one at the node (x_i, y_i) to zero at nodes (x_j, y_j), ($j \neq i$) (Sambridge *et al*. 1995). Linear interpolation of MOLA data was performed using the Interactive Data Language (IDL) software.

2b. Splining

Splining is a curve fitting method, which fits a least-cost mathematical function through observed data points (Hutchinson and Gessler 1994). Physically, it is similar to fitting a thin elastic sheet through the given points; the values on its surface become the interpolated data. Mathematically, a spline function $z(x, y)$ satisfies the following constraints:

$$z(x_k, y_k) = z_k \qquad \text{for all data } (x_k, y_k, z_k),\ k = 1, n \qquad (2)$$
$$(1 - t)\nabla^4 z - t\nabla^2 z = 0 \qquad \text{elsewhere}$$

where t is the tension, $0 \leq t \leq 1$. At $t = 0$, a minimum curvature solution is obtained, and $t = 1$ yields a harmonic solution (Smith and Wessel 1990). To minimize visible artefacts, a value of $t = 0$ was used.

For splining interpolation of MOLA data, the 'surface' program included in the Generic Mapping Tools (GMT) package was used, with a maximum of 10^6 iterations per computational cycle.

2c. Nearest Neighbour

In general, for every point of the output grid, a weighted average of n closest data points is performed. The weighting factor used most frequently is $1/r^2_i$, where r_i is the distance from the point being interpolated to the data point i (Eckstein 1989).

There is a nearest neighbour program available in the GMT package; however, it performs a sector-based neighbour search, which does not work well for MOLA data if a large n is used. Other widely available programs do not allow the user to rigorously define an output grid, or were found to have other problems. As a result, a simple nearest-neighbour procedure was implemented in the C programming language. A value of $n = 50$ and a weighting factor of $1/r^2_i$ was used for all tests.

2d. Natural Neighbour

The natural neighbour interpolation method has some features in common with linear and nearest neighbour techniques. In particular, it involves Delaunay triangulation and a weighted average, but it successfully avoids some of the problems of the aforementioned techniques. It differs primarily by the method of neighbour selection and the fact that weights are based on proportionate areas, rather than distances (Sibson 1981).

The natural neighbours of any point are defined as those to which the point is connected by the sides of Delaunay triangles, or equivalently, those in the neighbouring Voronoi cells. In the natural neighbour interpolation, a Voronoi diagram of existing data points is constructed. Subsequently, a new Voronoi cell is created about the interpolation point. If there are n natural neighbours of the interpolation point, the overlap of the new Voronoi cell with the original cells creates n new cells. The normalized area of the new cells is used as a weighting factor for the natural neighbours. A concise summary of this algorithm is available in Sambridge *et al*. (1995).

‘Natgrid’ is a natural neighbour interpolation package which is part of the ‘ngmath’ library distributed with NCAR Graphics. It contains both linear and non-linear implementations of the natural neighbour algorithm. For this study, the linear version was used, as it is roughly an order of magnitude faster, and no visual differences were discerned between the two approaches.

3. Qualitative analysis

Linear, splining, nearest neighbour, and natural neighbour techniques were used to interpolate MOLA data of the Korolev crater region (161–167 E, 72–74 N). The interpolated DEMs generated using the above methods were visually compared for visible artefacts. Figure 1 shows the visualization results.

[Insert figure 1 about here]

4. Quantitative analysis

For quantitative analysis, a DEM of a part of Iceland (15°45’ – 17°15’ W, 64°35’ – 66°10’ N), produced by the Icelandic Geodetic Survey, was used. The general concept is to sample data from it simulating MOLA data acquisition, interpolate that data, and then numerically compare the interpolated DEM to the original DEM (figure 2).

[Insert figure 2 about here]

For step 1 in figure 2, actual MOLA points were taken from the Korolev crater dataset. In step 2, elevations from the Iceland DEM were assigned to these MOLA points. These points were then used as input for the interpolation techniques.

[Insert figure 3 about here]

The output grids were created in low, medium, and high resolutions, corresponding to the resolutions of 1000, 250, and 82 pixels/degree on Mars. Figure 3 presents the interpolations to the medium (250 pixels/degree) grid. The resulting interpolated DEMs were then compared to the original Iceland DEM on a point-by-point basis, and the mean topography difference was calculated as follows:

$$\text{Mean topography difference} = \sum_{i=1}^{N} \text{abs}(elev2_i - elev1_i)\ /N \qquad (3)$$

[Insert table 1 about here]

[Insert table 2 about here]

A summary of the mean topography differences is shown in table 1. In addition, computational times were measured for each algorithm, and are presented in table 2.

5. Discussion

The results indicate that the natural neighbour algorithm consistently outperformed other techniques both quantitatively and qualitatively. Comparing the DEMs of the Korolev crater produced by the four interpolation methods (figure 1), the DEM generated by the natural neighbour algorithm clearly contains the fewest number of visible interpolation artefacts.

For the high resolution (1000 pixels/degree) interpolation of Icelandic data, the natural neighbour algorithm produced a realistic DEM with the fewest interpolation artefacts. It is also very promising that the overall lowest mean topography difference was achieved by the natural neighbour high resolution interpolation (table 1). The splining algorithm clearly did not work well for this resolution, and other methods had pronounced artefacts.

For the medium resolution (250 pixels/degree) interpolation (figure 3), the splining algorithm yielded a DEM that appears mostly artefact-free, but is highly inaccurate when analysed numerically. Both linear and nearest-neighbour methods result in a large number of visible artefacts and are quantitatively inferior.

The only exception is the low resolution (82 pixels/degree) interpolation results, which show a similar mean topography difference for all four algorithms and, with the exception of linear interpolation, have a roughly similar visual appearance. However, we anticipated that different interpolation methods would be best discriminated at high resolutions, and converge at low resolutions, so these results are not unexpected.

One disadvantage of the natural neighbour interpolation is that bad data points may not be recognized in the interpolated DEM, because they are made to look like natural features. Thus, it is important to eliminate bad data points prior to interpolation.

6. Conclusion

While further quantitative testing is desirable, it is clear that the natural neighbour algorithm yields excellent results when applied to MOLA data. Additional investigation in this area should include testing of the natural neighbour algorithm on other known DEMs and possibly combining it with the median observed topography technique. The current results indicate that natural neighbour should be the algorithm of choice when accuracy and realistic appearance are required, and splining can be used as a quick first-order interpolation technique. Also, interpolations with higher resolutions and better quality than those presented here are now possible with the recently released MOLA datasets MGSL2034–2054.

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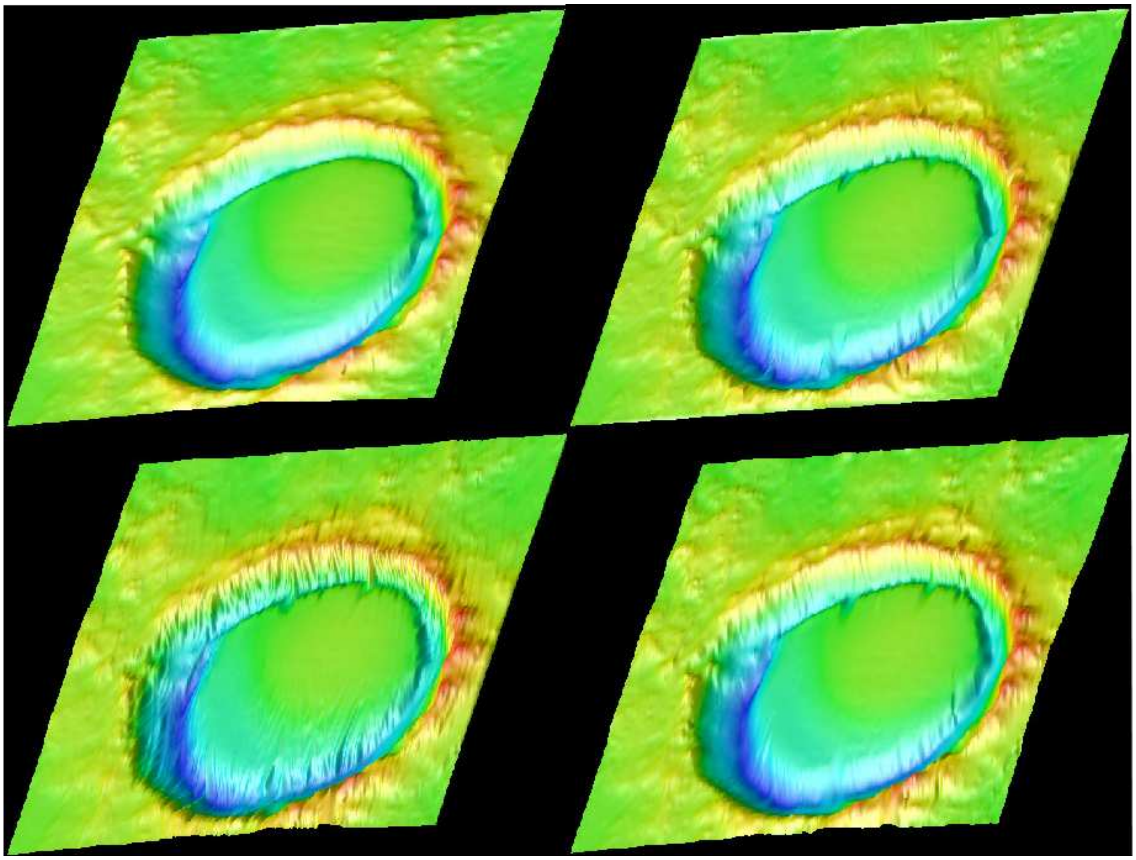

Figure 1. Interpolation techniques applied to the MOLA data of Korolev crater. Clockwise from top left: natural neighbour, linear, nearest neighbour, splining. The resolution is 200 pixels/degree.

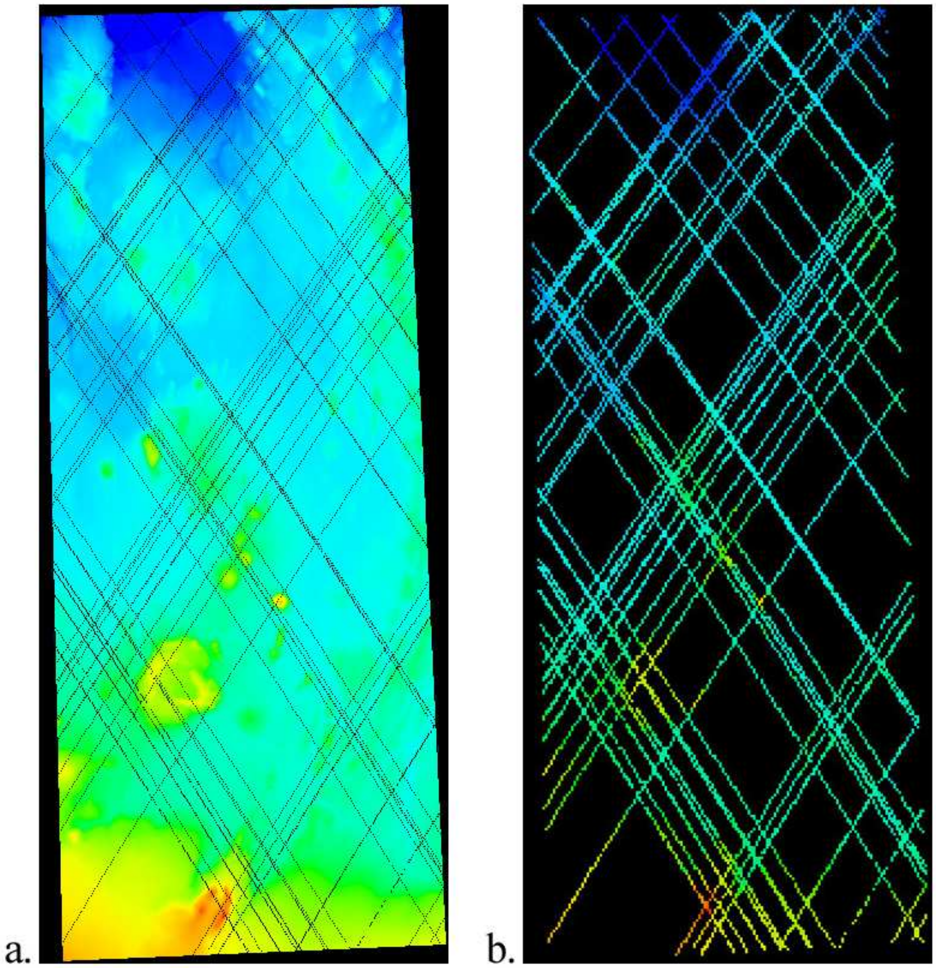

Figure 2. a) Random MOLA tracks are superimposed over the known DEM. b) Elevation values are obtained from the DEM at each MOLA point. An interpolated DEM is then created from these points and compared to the original DEM.

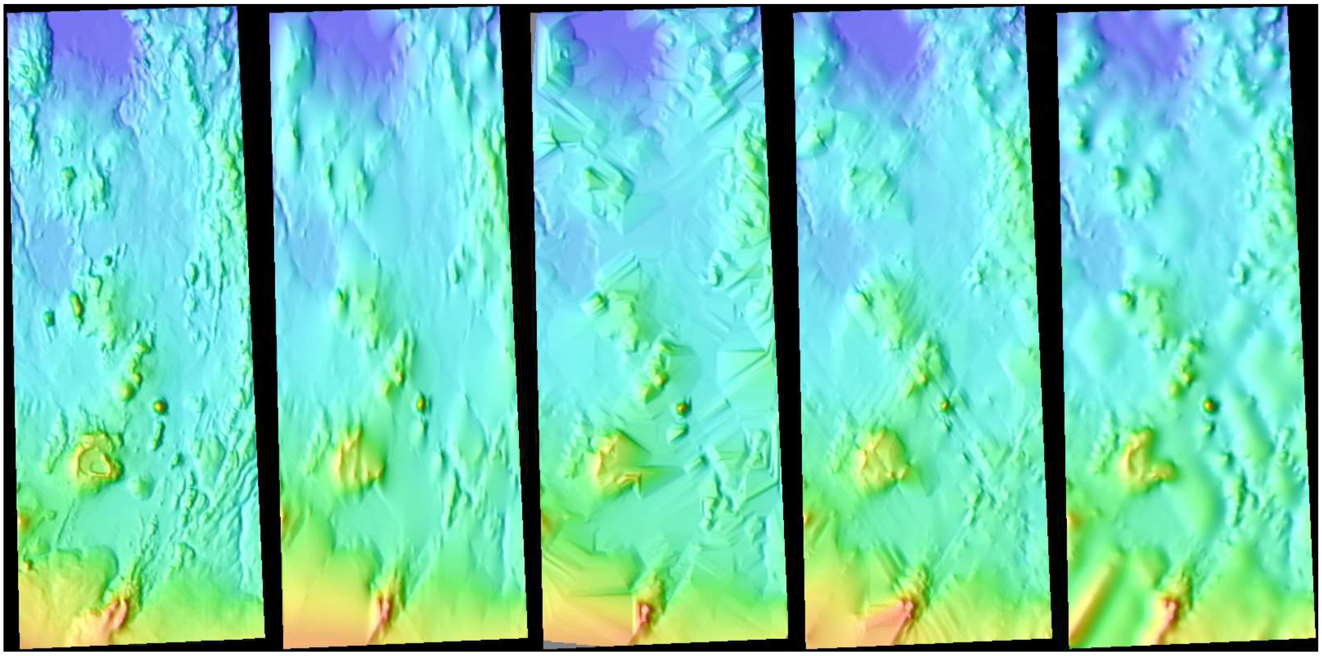

Figure 3. Medium resolution (equivalent to 250 pixels per degree on Mars) interpolations. From left to right: original DEM, natural neighbour interpolation, linear interpolation, nearest neighbour interpolation (with *n*=50), splining (minimum curvature, max. iterations per cycle = 10^6).

	High resolution interpolation (1000 pixels/degree)		Medium resolution interpolation (250 pixels/degree)		Low resolution interpolation (82 pixels/degree)	
	Mean topography difference	Standard Deviation	Mean topography difference	Standard Deviation	Mean topography difference	Standard Deviation
Natural neighbour	**25.51**	39.26	**27.23**	39.46	**36.22**	43.21
Linear	**34.03**	46.30	**32.43**	45.61	**34.64**	47.36
Nearest neighbour	**30.51**	43.81	**31.50**	44.30	**37.25**	48.29
Splining	**88.98**	116.49	**48.41**	92.05	**35.39**	48.64

Table 1. Summary of the quantitative analysis of interpolation techniques. All values are in metres. For the nearest neighbour technique, the number of nearest neighbours (n) was 50. For splining, 10^6 maximum iterations per cycle were used.

Interpolation algorithm	Execution time
Natural neighbour	02 hours 08 minutes 47.53 seconds
Linear	00 hours 00 minutes 05.70 seconds
Nearest neighbour	11 hours 09 minutes 11.58 seconds
Splining	00 hours 00 minutes 30.55 seconds

Table 2. Execution times for the interpolation of 80 732 data points to produce a DEM with a resolution of 200 pixels/degree.